Common Core Standards:
7.EE.3, 7.EE.4.a

Use Algebra to Solve Word Problems

Getting the Idea

One way to solve a word problem is arithmetically. Problem solving strategies can help you recognize the sequence of steps needed to solve a problem arithmetically.

Example 1

The art teacher has 6 packages of brushes and 8 single brushes. Each package has the same number of brushes. In all, he has 80 brushes for his students to use. How many brushes are in a package?

Strategy **Work backward to undo the sequence of operations in the problem.**

Step 1 Understand the problem.

There are 6 packages of brushes, each with the same number of brushes.

There are also 8 single brushes.

There are 80 brushes in all.

Step 2 Identify the first operation to undo.

The 8 single brushes are in addition to the packages of brushes.

Subtract 8 from the total of 80 to find how many brushes are in the packages in all.

$80 - 8 = 72$

There are 72 brushes in the packages in all.

Step 3 Identify the next operation to undo.

There are 72 brushes in the 6 packages. Since each package has the same number of brushes, divide to find the number in each package.

$72 \div 6 = 12$

Solution **There are 12 brushes in each package.**

Another way to solve a word problem is algebraically. First, define the variable in the problem situation. Next, write an equation to represent the situation. Then solve the equation using inverse operations and the properties of equality.

Inverse operations are operations that undo each other. Addition and subtraction are inverse operations, and multiplication and division are inverse operations. The properties of equality tell you that if you perform an operation to one side of an equation, you must perform the same operation to the other side of the equation to keep both sides equal.

When you solve an equation, you must get the variable by itself on one side of the equation. This is known as isolating the variable.

To solve the problem in Example 1 algebraically, first define the variable.

Let x represent the number of brushes in each package.

The equation $6x + 8 = 80$ represents the problem situation.

Solve the equation algebraically.

$$6x + 8 = 80$$

$6x + 8 - 8 = 80 - 8$ Use the inverse of addition. Subtract 8 from both sides.

$6x = 72$ Simplify.

$\frac{6x}{6} = \frac{72}{6}$ Use the inverse of multiplication. Divide both sides by 6.

$x = 12$

Notice that the solution is the same whether you solve the equation arithmetically or algebraically. When you solve the equation algebraically, you use symbols and properties to focus on the mathematics and simplify the process.

Example 2

Troy is 3 times as old as Mia's and Dan's ages combined. If Troy is 36 and Mia is 5, how old is Dan?

Strategy **Translate the problem into an algebraic equation. Then solve.**

Step 1 Translate the problem into an algebraic equation.

Let d represent Dan's age.

The total of Mia's and Dan's ages combined is $5 + d$.

Troy is 3 times the total. Troy is 36 years old.

$3(5 + d) = 36$

Step 2 Solve for d.

$$3(5 + d) = 36$$

$\frac{3(5 + d)}{3} = \frac{36}{3}$ Divide both sides of the equation by 3.

$5 + d = 12$ Simplify.

$5 - 5 + d = 12 - 5$ Subtract 5 from both sides to isolate d.

$d = 7$

Solution **Dan is 7 years old.**

Example 3

Ali was earning $15 an hour as a landscaper's assistant. She just received a 10% raise. What is Ali's new hourly rate?

Strategy **Solve the problem algebraically.**

Step 1 Translate the problem into an algebraic equation.

Let n represent Ali's new hourly rate.

Ali's old hourly rate is $15.

Ali's raise is 10% or $\frac{1}{10}$ of her old rate: $\frac{1}{10} \cdot \$15$.

Ali's new hourly rate is the total of her old rate and the raise.

$n = \$15 + \left(\frac{1}{10} \cdot \$15\right)$

Step 2 Solve for n.

$n = \$15 + \left(\frac{1}{10} \cdot \$15\right)$

$n = \$15 + \$1.50$

$n = \$16.50$

Solution **Ali's new hourly rate is $16.50.**

Example 4

Ron wants to place a painting that is $32\frac{1}{2}$ inches wide in the horizontal center of a wall that is $89\frac{1}{2}$ inches wide. How far from each corner of the wall should he place the painting for it to be centered?

Strategy **Solve the problem algebraically.**

Step 1 Translate the problem into an algebraic equation.

Let n represent the distance from each corner.

If the painting is centered, the distance from each corner plus the width of the painting is equal to the width of the wall.

$32\frac{1}{2} + 2n = 89\frac{1}{2}$

Step 2 Solve the equation for n.

$$32\tfrac{1}{2} + 2n = 89\tfrac{1}{2}$$

$32\frac{1}{2} - 32\frac{1}{2} + 2n = 89\frac{1}{2} - 32\frac{1}{2}$ Subtract $32\frac{1}{2}$ from both sides.

$2n = 57$ Simplify.

$\frac{2n}{2} = \frac{57}{2}$ Divide both sides by 2.

$n = 28\frac{1}{2}$

Step 3 Estimate to check that the answer is reasonable.

The wall is about 90 inches wide. The painting is about 30 inches wide.

$90 - 30 = 60$

For the painting to be centered, each edge of the painting will be about $60 \div 2$, or 30 inches from the corner of the wall.

Solution **Ron should place the painting $28\frac{1}{2}$ inches from each corner.**

Coached Example

The cost of renting a community pool for a party is $223 plus $20 for each person attending. Janeel has $500 to spend on a pool party. What is the maximum number of people that can attend Janeel's party if she does not go over her budget?

Let p represent the number of people attending the party.

Translate the problem into an equation.

_____

In the space below, solve the equation for p.

Interpret the solution.

What is the value of p? _____

Is it possible to have that number of people at a party? Why or why not?

_____ because _____.

You cannot invite part of a person to an event.

The value of p is between which whole numbers? _____

If I choose _____ as the whole-number answer, the cost will be greater than $500.

If I choose _____, the cost will be under $500.

If she spends no more than $500, the maximum number of people that Janeel can have at the party is _____.

Lesson Practice

Choose the correct answer.

1. Claire wants to place a mirror that is $18\frac{1}{2}$ inches wide in the center of a wall that is 31 inches wide. How far from each corner should she place the mirror for it to be centered?

A. $6\frac{1}{4}$ in.

B. $9\frac{1}{4}$ in.

C. $10\frac{1}{3}$ in.

D. $12\frac{1}{2}$ in.

2. Justin is 10 years less than half his father's age. If Justin is 12 years old, how old is his father?

A. 22

B. 24

C. 32

D. 44

3. The school media specialist has catalogued $\frac{2}{3}$ of the new books in a shipment. If she has catalogued 52 books, what is the total number of books in the shipment?

A. 35

B. 78

C. 104

D. 156

4. A video game is on sale for 30% off the regular price of $50. What is the sale price of the video game?

A. $20

B. $30

C. $33

D. $35

5. One school bus can seat 42 passengers. How many school buses will be needed to transport a total of 180 passengers on a trip to the state legislature?

A. 138

B. 10

C. 5

D. 4

6. Leah has three fewer than 4 times as many DVDs as Sonia has. Leah has 37 DVDs. How many DVDs does Sonia have?

A. 10

B. 11

C. 12

D. 36

7. In his bank account, Hal has $70 more than one-fifth the amount of money that Bob has in his account. Hal has $170 in his bank account. How much money does Bob have in his account?

A. $240

B. $350

C. $420

D. $500

8. Rudy wants to teach a hip-hop dance workshop. The cost of renting a dance studio is $109.50 plus $15 per person attending the workshop. Rudy can spend $318 at most to rent the space. What is the greatest number of people Rudy can allow to attend the workshop?

A. 13 people

B. 14 people

C. 28 people

D. 29 people

9. Sylvie bought 13 bagels and a container of cream cheese. Each bagel cost the same price. The cream cheese cost $2.95. Sylvie spent a total of $7.50.

A. Write an algebraic equation to represent the situation. Identify the variable.

B. What was the cost of each bagel? Explain your thinking.

10. In miniature golf, Janelle scored 5 strokes less than 150% of what Adam scored. Which situation could be true? Circle all that apply.

A. Adam scored 76 and Janelle scored 54.

B. Adam scored 71 and Janelle scored 54.

C. Adam scored 60 and Janelle scored 85.

D. Adam scored 52 and Janelle scored 73.

E. Adam scored 60 and Janelle scored 87.

F. Adam scored 50 and Janelle scored 70.

G. Adam scored 72 and Janelle scored 103.

11. Remy made cupcakes in batches of 12. There are 28 students in the class. Select True or False for each statement.

A.	Remy made 2 batches and had 4 cupcakes left over.	○ True	○ False
B.	Remy made 3 batches and had 4 cupcakes left over.	○ True	○ False
C.	Remy made 3 batches and had 8 cupcakes left over.	○ True	○ False
D.	Remy made 4 batches and had 20 cupcakes left over.	○ True	○ False

12. Grant is 5% taller than he was last year. How tall is Grant if his height last year was as given below? Circle each correct height.

56 inches:	56.8 in.	60 inches:	63 in.	68 inches:	68.8 in.
	58.8 in.		66 in.		71.4 in.
	61.6 in.		69 in.		74.8 in.

13. Marnel ran 3 miles less than twice as far as James. Marnel ran 7 miles. How far did James run, j? Use values from the box to complete the equations and solve the problem.

_____ j − _____ = 7

2 × _____ = 7 + _____

_____ = 10 ÷ _____

j = _____

j
2
3
5

14. To get to the amusement park, 14 people need to have a seat in a car. Each car holds 4 people. Is each given number of cars enough to hold all the people? Select Yes or No.

A. 3 ○ Yes ○ No

B. 4 ○ Yes ○ No

C. 2 ○ Yes ○ No

D. 6 ○ Yes ○ No

15. Elise paid $\frac{2}{3}$ as much for a sweater as she did for a pair of shoes. How much did Elise pay for the sweater if the shoes cost each given amount? Circle each correct amount.

Shoes cost \$60: Sweater cost

\$20
\$40
\$90

Shoes cost \$75: Sweater cost

\$25
\$50
\$112.50

Shoes cost \$42: Sweater cost

\$14
\$28
\$63